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Curve Style Analysis in a Set of Shapes

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Abstract

The word "style" can be interpreted in so many different ways in so many different contexts. To provide a general analysis and understanding of styles is a highly challenging problem. We pose the open question "how to extract styles from geometric shapes?" and address one instance of the problem. Specifically, we present an unsupervised *algorithm for identifying* curve styles *in a set of shapes. In our setting, a curve style is explicitly represented by a mode of curve features appearing along the 2D silhouettes of the shapes in the set. Unlike previous attempts, we do not rely on any preconceived conceptual characterizations, e.g., via specific shape descriptors, to define what is or is not a style. Our definition of styles is* data-dependent*; it depends on the input set but we do not require computing a shape correspondence across the set. We provide an* operational definition *of curve styles which focuses on* separating *curve features that represent styles from curve features that are content-revealing. To this end, we develop a novel formulation and associated algorithm for style-content separation. The analysis is based on a* feature-shape association matrix *(FSM) whose rows correspond to modes of curve features, columns to shapes in the set, and each entry expresses the extent a feature mode is present in a shape. We make several assumptions to drive style-content separation which only involve properties of, and relations between, rows of the FSM. Computationally, our algorithm only requires row-wise correlation analysis in the FSM and a heuristic solution of an instance of the set cover problem. Results are demonstrated on several datasets showing the identification of curve styles. We also develop and demonstrate several style-related applications including style exaggeration, removal, blending, and style transfer for 2D shape synthesis.*

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling —Curve, surface, solid, and object representations

1. Introduction

It seems possible to speak of styles of all human designs and endeavors. The elegance and distinctiveness of styles draw our attention to them, making them interesting subjects of study. Styles in speech, writing, arts, fashion, architecture, etc., are constantly analyzed, imitated, and invented [Ken82, LRF04, Chi07]. In this paper, we are interested in an analysis of styles pertaining to the geometry of shapes.

The open question we pose is "how to extract styles from shapes?" Even when confined to a study of shape geometry, the notion of style can still be interpreted in so many different ways and contexts that it is likely impossible to provide an analysis of all styles of all shapes. In this paper, we are interested in shape styles that are *local* and *visually explicit*. Specifically, we focus on *curve styles*, styles of a shape that are explicitly revealed in certain segments of its exterior 2D silhouette, e.g., see Figure 1. Silhouette curves are highly effective in conveying shapes and various shape styles are well presented in the silhouette profiles. Moreover, curves are easy to parameterize with simple boundaries to facilitate blending and stylized shape synthesis.

Our specific question then is "how do we judge whether a curve segment represents a style or not?" A supervised learning approach would rely on training data to provide a collection of style examples and resort to pattern matching to identify styles in test shapes. In this paper, we would like to perform style analysis in an *unsupervised* setting. This is more challenging since there is no prior knowledge about the styles being sought. Furthermore, we believe that it would be difficult, if not impossible, to provide a general *conceptu-*

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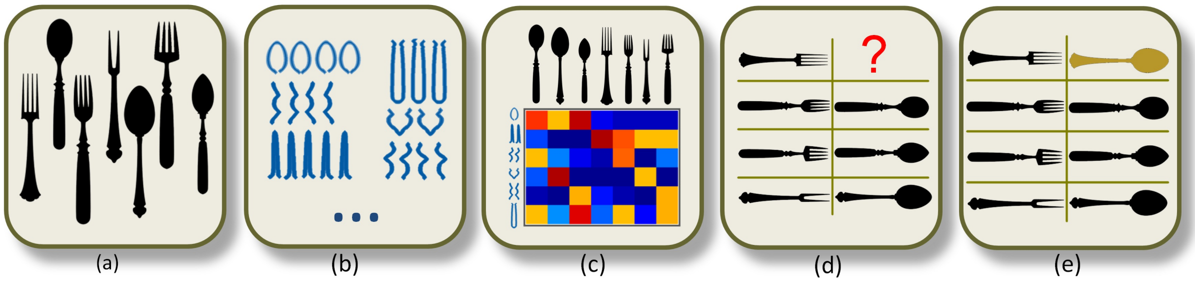


Figure 1: Curve style analysis in a set of shapes. (a) Input set with diverse content and style. (b) Extracted curve features clustered into feature modes. (c) Construction of feature-shape association matrix (FSM) whose rows are consolidated feature modes and whose columns correspond to the set of shapes. (d) Style-content table resulting from style-content separation. (e) Style transfer applied to fill the blanks.

al or theoretical definition for curve styles, i.e., a definition that precisely characterizes the "intrinsic essence" of curve styles. In particular, when a curve feature is given in isolation, one can hardly judge whether it is a style curve or not. However, when the feature is presented in context one may be able to make better judgements. In this paper, we aim for an *operational* definition [STL04] of curve styles that is *data-dependent*. That is, we define whether a curve segment represents a style or not in the context of a set of extracted curve segments and identify the style segments by a computational process, the operation, that separates styles from non-styles.

Our operational definition of curve styles is built on several basic premises. First, a stylistic curve segment should be an interesting feature in its own right. Correspondingly, our style analysis starts with feature extraction. Second, a broader context is needed to judge whether a curve feature represents a style or not. In this work, we utilize a *set* of shapes as input and identify styles from a set of detected curve (segment) features. Third, and most important, given the curve features extracted from the set of shapes, the focus of our operational definition is on *style-content separation*.

We classify the curve features into style-revealing, content-revealing, and background features. Similar to our treatment of curve styles, we do not rely on any hard-coded list of characteristics to identify content. Rather, we rely on an algorithm to separate style features from the content and background features. We only require that curve content can be revealed by curve segments as well.

We develop a novel formulation and associated algorithm to perform style-content separation over a set of curve features extracted from a set of shapes. The analysis is based on a *feature-shape association matrix* (FSM) whose rows correspond to modes of curve features, columns to shapes in the set, and each entry expresses the extent a *feature mode* is present in a shape; see Figure 2 for an example. Note that a feature mode is nothing but a group of similar curve (feature) segments. Each row of the FSM then corresponds to a style feature, a content feature, or a background feature.

We make several assumptions (Section 2) to guide our style-content separation. The assumptions only involve properties of, and relations between, rows of the FSM. Computationally, our algorithm only requires row-wise correlation analysis in the FSM and a heuristic solution of an instance of the set cover problem.

Related works. Most works on style-content separation have followed the supervised learning approach and are

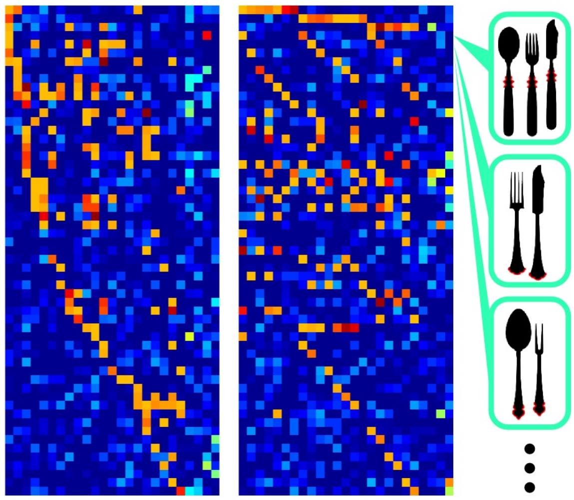


Figure 2: Humans can easily spot curve styles from a set of shapes. In an unsupervised setting, a machine only sees the association between shapes and extracted low-level features, shown as a seemingly random matrix on the left. Our style-content analysis operates purely on such a matrix and discovers the patterns therein which reveal both contents (top three rows in the second matrix) and styles (rows which follow with a few styles shown on the right).

based either on parameterized models [TF00, WFH07] or statistical modeling using PCA [BV99, BH00]. Both types of methods require correspondence across the input set, which is difficult to compute, and the shapes must belong to the same class. The resulting styles are global and latent in nature; the styles we identify are local, decorative, and visually explicit. Our analysis is unsupervised; it does not require correspondence and operates across contents.

The recent work of Xu et al. [XLZ*10] on style-content separation is unsupervised and correspondence-free, like ours. The two works both produce a style-content table and perform style transfer to fill blanks in the table. However, their work is designed for a specific shape style given by part scales. In our work, the styles are unknown and must be discovered through set analysis.

There has been a large amount of work on mining frequent [HCXY07] or unusual [KNT00] patterns in a set. The styles we identify are judged neither by their frequency nor by them being outliers. Past works on curve styles including [HOCS02, FTP03] learn the styles from a given set of curves for stylistic line drawings. In contrast, our work focuses on curve styles of a set of silhouette contours.

There has been a tremendous amount of work on content analysis of shapes, particularly for shape retrieval [TV08]. In all of these works, shape contents are characterized by certain shape descriptors. Our curve feature extraction, clustering, and the use of the FSM may remind one of the "bag of words model" (BoW model) in computer vision [FFP05], which has been widely used for object recognition, image classification, and more recently, 3D shape retrieval [BBOG11]. Our work extracts shape features, but differs significantly from previous works on BoW model in the way these features are utilized as well as the target problem. Our work does not address the shape classification or discrimination problem. The histogram of curvature descriptor is applied only to group similar curve segments per shape to construct the FSM, not to classify or recognize shapes.

Most recently, Doersch et al. [DSG*12] extract image features that best characterize the city of Paris. One can consider this work to be also about style analysis, namely, it attempts to identify the distinctive styles of Paris, in the form of local image features. The same methodology should be applicable to characterize other distinguishable image or shape classes. However, their analysis technique is supervised, with a clear goal of identifying characteristic local features.

Contribution. Our main contribution is a novel formulation based on FSM analysis and an associated algorithm for style-content separation. The style-content analysis leads to an unsupervised method for extracting curve styles from a set of 2D shapes. Our analysis does not rely on any shape descriptors or other conceptual characterizations to define curve style or content; it also does not require any shape correspondence across the set. It allows for the handling of shapes across content classes and possessing generic curve styles, as long as the styles are visually explicit, local, and decorative in nature. In addition to showing our algorithm's ability to identify curve styles in a set of shapes, we also demonstrate that the identification of curve styles facilitates several style-related 2D shape modeling tasks.

2. Assumptions

In general, a curve style or content is not specific to one curve segment but may be shared by several segments. Hence we cluster extracted curve features from the input set of silhouettes to form what we call the feature modes. Each feature mode can be content-revealing, style-revealing, or neither, e.g., it is a background feature. In Figure 2, for example, the curve features (showed in red) in each green box belong to one feature mode, revealing either style or content. A baseline assumption we make on the input set is:

I: The input set contains diverse and significant contents, as well as diverse and significant styles.

This is translated into a consideration on the number of shapes in the set that are "covered" by a content-revealing or style-revealing feature mode. This number can be neither too large (not diverse) nor too small (not significant). This assumption leads to a pre-filtering of background features, and is also used for content identification. Here we say a shape is covered by a feature mode if some segment belonging to the feature mode appears in the shape.

Without any training data or other prior knowledge to indicate what is style and what is content, it is difficult for a machine to separate the two types of features. Unlike humans, a machine only "sees" the subsets of shapes that are covered by the various feature modes. Our goal is to allow the machine to extract curve styles based solely on this information. To this end, we make three more assumptions on the input set:

II: Each shape in the set belongs to a *unique* content class.

This is a weak prior. An equivalent way of stating it is that the subsets covered by the curve contents form a *non-overlapping partition* of the input set. We do not assume the same coverage property on style features. For example, a shape in the set can be style-less.

III: There are more style features than content features in the input set.

This is perhaps the most debatable assumption among all. The intuition behind it is that in a sufficient large set of shapes, styles should be expected to be more *distinctive* than contents. This distinctiveness is correlated to the feature count. The more the features are, the more distinctive each feature is. It is certainly possible for this assumption to be broken, which may lead to a confusion between what defines style and what defines content. However, the other assumptions would still allow us to *separate* the two classes of features. The binary decision of which class is regarded as style can be made according to additional criteria.

IV: Style and content tend to be *uncorrelated*.

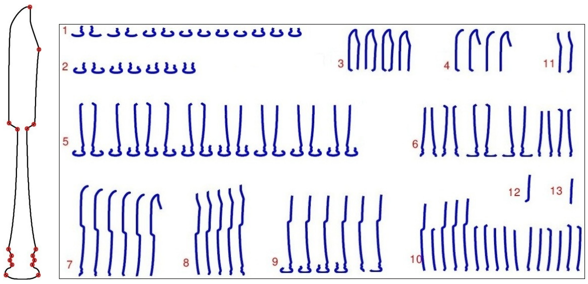
To be more precise, note that in our formulation, a style or content is represented by a row in the FSM. Then the above assumption can be restated as: two rows where one represents a style and the other a content have low correlation; see Equation (3) for the definition of correlation we adopt. This assumption is reminiscing of the classical notion of style-content separation [TF00] which considers styles as *orthogonal* to content. Also, the assumption is consistent with the observation that styles often cross content class boundaries in real-world datasets.

It is important to note that none of the above assumptions provide a conceptual definition of what a style or a content feature is. We do not believe it is possible to offer such definitions in general without raising objections since it is too subjective of a matter and depends on the application context. All our assumptions speak of properties of, and relations between, subsets covered by the feature modes. We show in Section 5 that style-content *separation* is achievable under these assumptions.

3. Overview

The input to our algorithm is a set of 2D shapes represented by their silhouettes. Our analysis only accounts for exterior silhouettes; interior contours or holes in the 2D object representations are ignored. To model content and style coverage of the set of shapes, we introduce the *feature-shape association matrix* or *FSM* whose rows correspond to feature modes, columns to shapes in the set, and each entry expresses the extent a feature mode is present in a shape. Our algorithm proceeds in three steps (see Figure 1):

1. **Per-shape feature extraction:** For each shape, we extract a set of feature curve segments and then cluster them to remove redundancies caused by curve similarity, noise, and in particular, repetition of curve patterns. Note that curve styles tend to contain such pattern repetitions. The result of this step is a set of feature modes per shape.
2. **Feature mode consolidation and FSM construction:** We form the initial FSM by defining its rows using the per-shape feature modes. By definition, the FSM reveals the extent each feature mode appears in other shapes in the set as well. We consolidate the set of feature modes by clustering them based on correlations among the rows of the initial FSM. Each cluster of highly correlated rows is condensed via simple averaging into a single row representing a feature mode across the set of the shapes. These feature modes form the rows of the new consolidated FSM for style-content analysis.



(a) (b)

Figure 3: Per-shape curve feature extraction. (a) 16 curvature extrema (red dots) are detected along the contour of a knife, which are later used to extract 93 curve segments. (b) 13 per-shape feature modes are generated by clustering all these curve segments.

3. **Style-content separation:** From the FSM, we filter background features using Assumption I, identify content features, and finally extract style features.
   a. *Content identification:* We define content features as the set of rows in the FSM whose covered subsets form the *smallest* (in number) non-overlapping partition of the whole set of shapes. In Section 5, we provide arguments to justify this definition.
   b. *Style identification:* Having identified the content rows in the FSM, we sort the remaining rows according to their correlation with the content rows. Based on Assumption IV, rows having low correlations are identified as style features. Any leftover background or content features are filtered out as well.

The outcome of the analysis is a separation between styles and contents identified from the input set, resulting in a *style-content table*. Each row in the table represents a detected curve style and content groups are arranged into columns; see Figure 1(d). The table allows for style transfer for the synthesis of new stylized shapes, as shown in Figure 1(e) and Figures 14-16. We also describe and demonstrate several other style-related applications including style removal, blending, and exaggeration.

Again, it is important to note that our style-content analysis does not need any shape correspondence between the input shapes; this is unlike most previous works developed for style-content separation.

4. Feature analysis

In this section, we describe how the curve feature modes are extracted from the input set of silhouette contours and then consolidated to form the FSM for style-content analysis.

Per-shape curve segment extraction. We start with intra-shape feature extraction to obtain a set of curve segments

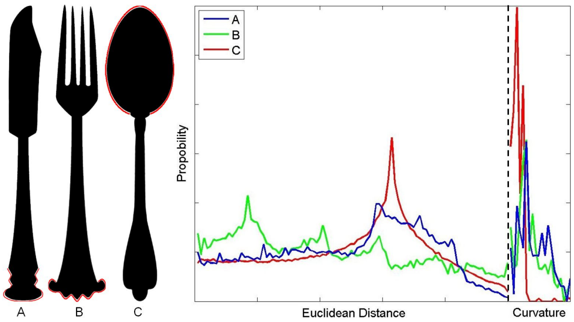


Figure 4: Shape distributions (100 bins) and curvature histograms (20 bins) are used to define the dissimilarity measure among curve segments (highlighted in red on the left).

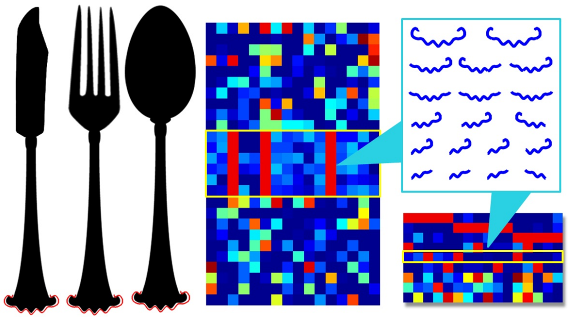


Figure 5: A group of highly correlated feature modes in the initial FSM (between yellow lines in middle matrix) is condensed into a single feature mode in the consolidated FSM.

per input silhouette. Along a given silhouette, we estimate curvature and return all curve segments delimited by a pair of curvature extrema, but with a length upper bound set at half of the length of the entire contour (see Figure 3a). Since the input silhouette curves from our test sets are generally quite clean, we estimate curvatures using discrete finite differences. In the case of noisy curves, more robust curvature measures such as integral invariants are more appropriate.

Per-shape feature modes. Given the set of curve segments obtained for each shape, we cluster them to remove redundancies caused by curve similarity, noise, and in particular pattern repetition. The result is the initial set of feature modes, one per cluster per shape (see Figure 3b).

The clustering scheme we employ is the complete linkage hierarchical clustering [ELL09]. The clustering process stops once the minimum complete distance between any pair of clusters exceeds a distance threshold τ_δ. In all the experiments, we use a conservative threshold $\tau_\delta = 0.1$ which would lead to early stopping and hence possibly redundant clusters . In the next step, feature mode consolidation, such redundant clusters are dealt with across the set of shapes.

The distance measure used in the clustering process is a dissimilarity measure between curve segments; it is built upon the shape distributions [OFCD02] and the curvature histogram descriptors. Histogram-based descriptors are more robust against scale differences and replication of patterns which are both common for curves possessing the same style. The dissimilarity δ_{ij} between two curve segments i and j is given by

$$\delta_{ij} = \sqrt{\|H_i - H_j\|_2^2 + \|K_i - K_j\|_2^2}, \qquad (1)$$

where H_i and K_i are the shape distribution and curvature histogram of curve segment i, respectively. Figure 4 illustrates the shape distributions and curvature histograms of three curve segments. The shape distribution here is a 2D analogue of the D2 shape distribution in [OFCD02], which measures the Euclidean distance between two random points on the curve segment. Both shape distribution and curvature histogram are aligned by the maximum sample values to account for differences in scale.

Initial FSM. Let there be n shapes in the input set and m clusters (initial feature modes) obtained from the previous step. We define the initial FSM A as the $m \times n$ matrix with

$$A_{ij} = exp(-d_{ij}^2/\theta^2), \text{where } d_{ij} = \min_{s \in S_j} \frac{\sum_{t \in F_i} \delta_{st}}{|F_i|}. \qquad (2)$$

Here F_i and S_j denote the sets of curve segments from cluster i and shape j, respectively, and the Gaussian width $\theta = 0.2$. In other words, d_{ij} is the minimal mean dissimilarity between any curve segment s from shape j and all the curves in feature mode F_i. Here m is the total number of per-shape feature modes, which is typically 10-30 per shape, over all the shapes in the data set.

Note that we do not define A_{ij} as a membership indicator, but use curve similarity to compensate for any random errors which may arise from the clustering. That is, A_{ij} does not indicate whether any segment in F_i belongs to shape j but rather measures the "extent" a segment in F_i appears in j.

Consolidated FSM. The initial FSM now contains association information between any per-shape feature mode and any shape. Thus it allows an inter-shape feature analysis. The purpose of feature mode consolidation is to arrive at a condensed set of feature modes to reflect contents and styles in the input set. A content-revealing (respectively, style-revealing) feature mode is a mode of features shared by the subset of shapes possessing that content (respectively, style). The subset of rows in A that form such a feature mode are those that are highly correlated; see Figure 5.

We consolidate the initial set of feature modes by clustering the rows of the initial FSM A, where the distance measure employed arises from row-wise correlation analysis in A. Regarding each row of A as observations of a random variable, we define the correlation between a pair of rows

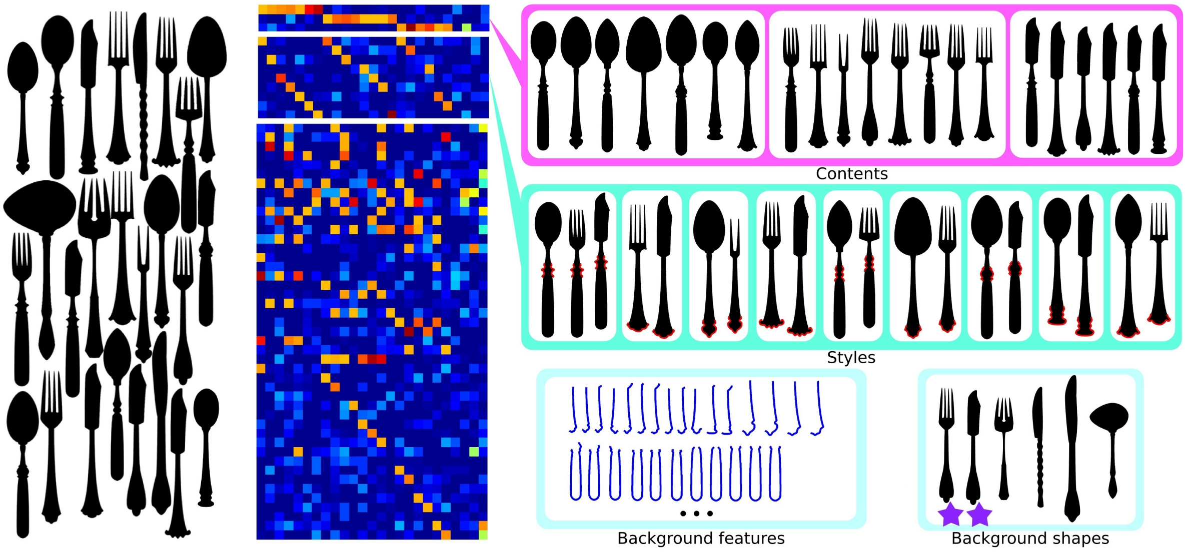


Figure 6: Style-content analysis on a set of cutlery shapes (left). Rows and columns of the FSM are re-ordered to clearly indicate content (top), style (middle), and background features (bottom). Right: shapes and curves that correspond to these features. Only two background feature modes are displayed. Shapes marked by the purple stars show an undetected curve style.

(X,Y) based on the Pearson product-moment correlation coefficient R,

$$corr(X,Y) = \begin{cases} R(X,Y) & \text{if } R(X,Y) \geq 0, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

where

$$R(X,Y) = \frac{Cov(X,Y)}{\sqrt{Cov(X,X)Cov(Y,Y)}}, \tag{4}$$

and $Cov(\bullet,\bullet)$ is the covariance between two sets of data.

We define a distance measure between two rows A_i and A_j of A as $dist(A_i,A_j) = 1 - corr(A_i,A_j)$. The clustering of the rows is again carried out by complete linkage hierarchical clustering with a stopping condition $\tau_{dist} = 0.1$. We compact the initial FSM A into a consolidated FSM $\hat{A}$, in which each row corresponds to a cluster. The row, a (consolidated) feature mode, is obtained by a simple averaging of the rows from A that belong to the cluster.

Note that by using row correlations for clustering, as a consequence of Assumption IV from Section 1, a row representing a style and a row representing a content are less likely to be clustered or consolidated into a single row in $\hat{A}$; this is certainly desirable as the goal of our analysis is style-content separation. Moreover, the use of a conservative threshold for this second clustering step also avoids overclustering of the rows.

5. Style-content separation

Our style-content analysis operates solely on the FSM $\hat{A}$. Before executing style-content separation, we first pre-filter the rows to remove some background features. The "diversity" assumption (Assumption I) on style and content implies that they cannot be too dominant in terms of the number of shapes they cover. We use a conservative threshold of 50% to filter out any row that covers more than half of the shapes in the set, where a feature (row) i is said to cover a shape (column) j if $\hat{A}_{ij} > \tau_{coverage} = \exp(-1/4)$.

Content identification. After the pre-filtering, we extract rows from $\hat{A}$ that we deem to correspond to content features. In each row i, we flag all the *peaks*, entries along the row whose values exceed $\tau_{coverage}$. Note that these peaks indicate coverage of shapes by the feature mode in row i. Denote by $S_i \subseteq U$, where $U = \{1,2,\ldots,m\}$, the set of shape indices corresponding to the flagged peaks in row i. Our content detection scheme seeks to find the smallest (in cardinality) collection of S_i's that form a non-overlapping partition of U. The problem is NP-complete since it can be shown that the set cover problem [Kar72] reduces to it.

We now justify the above criteria for content identification based on the assumptions we made from Section 1. Without loss of generality, let $S_{i_1},\ldots,S_{i_k}$ form the smallest non-overlapping partition of U. Then we claim that all the rows i_1 to i_k correspond to content-revealing features. Assuming otherwise, then there must be a subset U' of U that is covered by a set R_1 of style rows. However, by Assumption II, U' must also be covered by a set R_2 of content rows. Further,

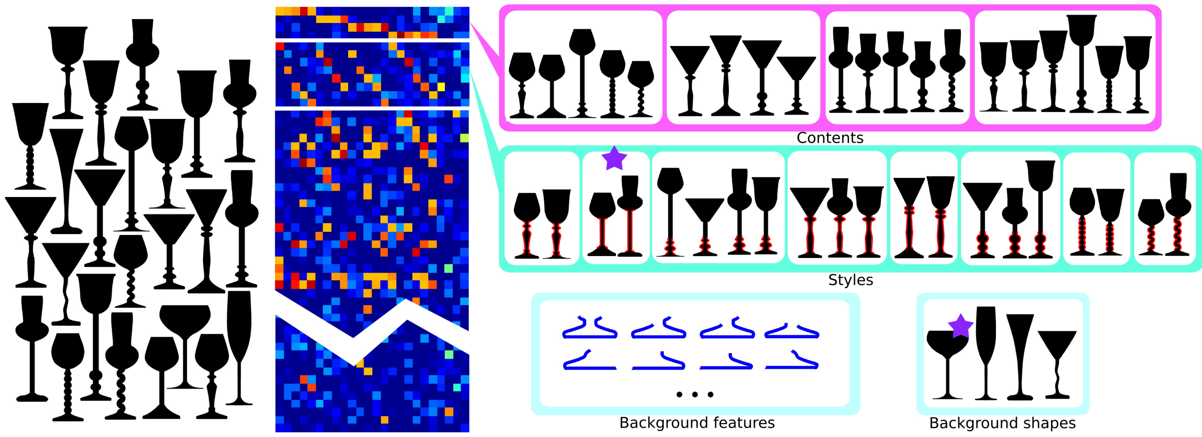


Figure 7: Style-content analysis on a set of goblets. Purple stars point to "non-styles" identified and a shape sharing the feature.

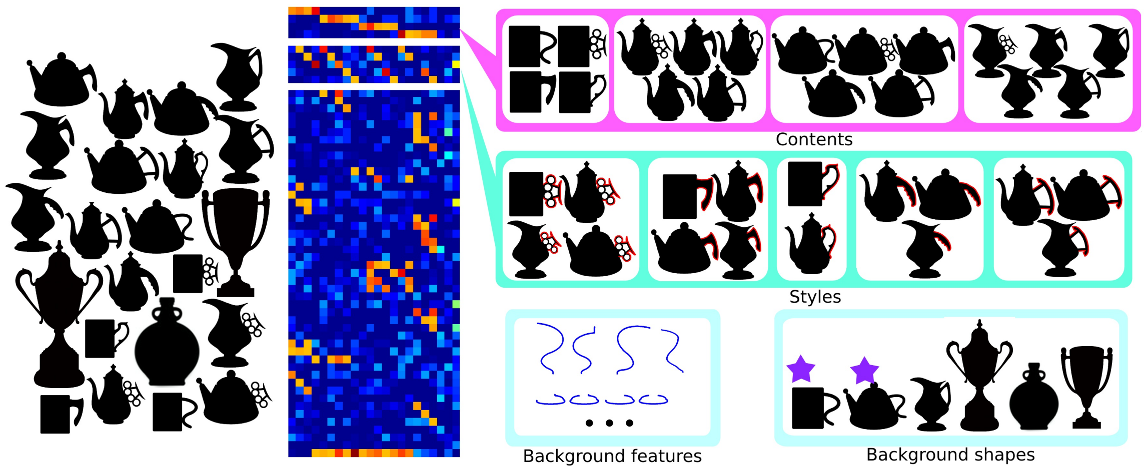


Figure 8: Style-content analysis results on a set of teapot and cup shapes. Purple stars point to an undetected style.

by Assumption III, by replacing R_1 by R_2, since $|R_2| < |R_1|$, the total number of rows, the size of the non-overlapping partition, would decrease, contradicting with our assumption. This concludes our justification.

There may very well be heuristic solutions to the non-overlapping set cover problem. However, since the number of rows in $\hat{A}$ arising from all of our datasets is always rather small, we simply resort to exhaustive search. In practice, due to the existence of noise in the data, e.g., background shapes which do not belong to any significant content groups, we "soften" the search criteria, only requiring the non-overlapping collection of S_i's to cover $\tau_{content} = 80\%$ of the shapes in the input set. Suppose there are multiple solutions of the set cover problem and the best solution has cardinality z. Among those solutions whose cardinality is no more than $z+1$, we choose the solution that has the smallest variance over the cardinalities of S_i's. Assumption I implies that the cardinality of S_i can be neither too large (not diverse) nor too small (not significant), which can be translated into the cardinalities of S_i's should have a small variance.

Indeed, those remaining shapes not covered by the collection from our solution are regarded as background shapes.

Style identification. With the set C of content features identified, we sort the remaining rows by their correlation with C. Specifically, we define the *style score* of row j by

$$\sigma_j = 1 - \max_{i, \hat{A}_i \in C} \frac{corr(\hat{A}_i, \hat{A}_j)}{q}, \qquad (5)$$

where q is the number of content features (rows) in C whose boundary $\hat{A}_j$ crosses. Top-ranked rows are deemed to correspond to style features and those rows scored below $\tau_{style} = 0.95$ are regarded as corresponding to background features. This style identification scheme is motivated by Assumption IV. Finally, any shape not covered by any identified style is regarded as a background shape as well. Thus, we require a non-background shape to possess both an identified content and one or more identified styles.

6. Style extraction results

Results in this section mainly demonstrate the ability of our algorithm to identify intuitive curve styles and contents from a set of shapes. Three sets of results are given in Figures 6-8, all obtained using the same set of threshold parameters. Additional analysis and application results can be found in the supplementary material.

Datasets and data source. Each set of shapes in our experiment possess diverse contents and diverse styles. Although there is no technical requirement on what type of contents can be included in a set, the analysis and application results (see Section 7) are more meaningful when the shapes fall in a similar category. The majority of the silhouette contours in the test sets were obtained from on-line repositories including bigstock.com and shutterstock.com. However, these readily available shapes often do not come in sufficient numbers to form non-trivial content groups. To this end, we added more shapes into the datasets by manual part transplantation to ensure that the content groups have non-trivial sizes (more than 25 shapes per group). Note that we have not synthesized any new curve styles; all curve styles in the sets were found in the on-line repositories.

Style-content separation and identification. To better illustrate the style-content separation results we obtain from the FSM, we re-ordered its rows and columns. Content features (rows) are on top, followed by style features sorted by their style score, and then background features. Column-wise, the shapes are arranged in content groups with background shapes listed to the right; see Figures 6-8. Shapes and curves correspond to these features are displayed to the right; only two background features are shown in each case. While we can generally observe intuitive identification of styles and contents, our analysis results can be imperfect or at least debatable. Figures 6 and 8 show possible undetected styles (marked by purple stars). In Figure 7, a rather non-stylish pair of goblets had a high style score and one of the identified background shapes actually possesses the same "style" (similarly marked).

Performance. The running time of our algorithm is dominated by the number of curve features detected for a given set of shapes, since clustering curve features is expensive and scales quadratically (typically 30-200 curve segments were extracted for each shape). Our current non-optimized implementation typically takes 10-20 minutes for most sets that include around 30 shapes.

Note that we apply a two-step clustering scheme, including per-shape feature modes clustering and FSM consolidation clustering, instead of a global clustering on the set of all

Figure 9: Our algorithm fails to identify the curve styles in the cutlery set shown in this figure since the set doesn't contain significant styles. Also our curve dissimilarity measure fails to cluster all the knife heads as one feature mode.

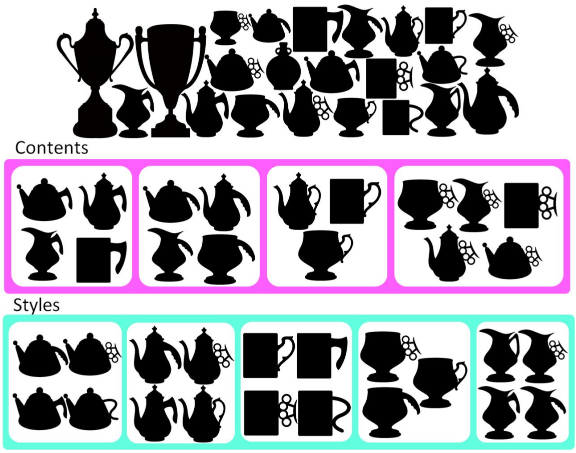


Figure 10: Our algorithm interprets styles as contents in the set of teapots, because this set is not general enough to reflect the relationship between content and style (breaking Assumption III).

curve segments from all shapes. Suppose shape j produces n_j curve segments, and since clustering has quadratic complexity, the performance of feature modes clustering can be improved from $(\sum n_j)^2$ to $\sum n_j^2$. Note when the number of rows in the intial FSM increases, the complexity in the second step clustering is increased as well. However due to the scale of FSM being much smaller comparing to the scale of all curve segments, the improvement can become significant when the number of shapes and n_j are both big.

Failure case. Our algorithm would fail to identify curve styles in a natural way if the given set does not satisfy our current assumptions. Figure 9 shows a cutlery set that contains significant contents but no significant styles (breaking Assumption I). Although it seems clear for a human that this set contains three major contents, our relatively simple curve dissimilarity measure in Equation 1 fails to cluster all the knife heads as one single feature mode. This is because some handles look very similar to the knife heads and thus violate the clustering. In this case, more sophisticated dissimilarity measures such as the Fast Marching Method [FB03] can be applied, however this method would increase the computational cost significantly. This set also shows an example of Assumption (III) at work, i.e., the number of style varieties is expected to exceed that of content varieties in the set.

For the teapot set in Figure 10, our algorithm interprets styles as contents, thus a confusion. This is mainly because this small input set breaks Assumption III. However, our algorithm is still able to return two sets of feature modes from the teapot set, one content and the other style. The binary decision of which is style and which is content can be made by additional assumption(s) or by the user.

7. Applications

Analyzing curve styles allows identifying the style curves in a shape and developing several style-based applications. On a single shape, one can remove or exaggerate an identified style curve. In a set of shapes, based on style-content separation, one can blend between the styles of two shapes or transfer the style from one shape to another. The latter also enables changing the style of whole objects as well as synthesizing missing content with a given style.

Style removal. Silhouette curve styles typically correspond to high-frequency contents. Thus, removing a style is achievable via smoothing. However, simply smoothing the entire contour of a shape not only removes the style but also distorts other parts of the shape. By identifying specifically the style curves, one can concentrate on their removal. Since style segments are typically feature-rich, potential applications of style removal include the elimination of such extraneous feature points, assisting feature-driven analysis tasks such as outline segmentation or finding correspondence.

Our approach to style removal is to delete the segment containing the style curve from the outline contour and then smoothly blend in a style-free replacement. The key idea is to replace the style curve by a smoothed version which maintains the base shape of the curve. The base curve is defined using implicit Laplacian smoothing [SCO04] of the style curve. Assuming that the style curve is defined by a piecewise-linear set of points $V = \{v_1, v_2, ..., v_n\}$, the base curve is the curve that minimizes the quadratic energy:

$$\|\Delta V'\|^2 + \sum_i w^2 \|v'_i - v_i\|^2, \qquad (6)$$

where $V' = \{v'_1, v'_2, ..., v'_n\}$ are the vertices of the base curve after style removal. The first term removes geometry details along the normal directions using the Laplacian; the second term preserves the base shape and is controlled by the positional weights w. If L is the $n \times n$ Laplacian matrix then this energy is minimized by solving the following linear system:

$$\begin{bmatrix} L \\ wI \end{bmatrix} V' = \begin{bmatrix} 0 \\ wIV \end{bmatrix}. \qquad (7)$$

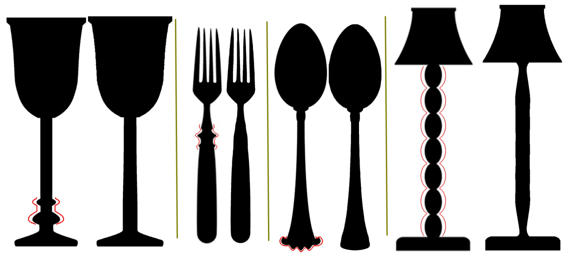


Figure 11: Style removal: the identified style-curves (red) are replaced by a smoothed style-less segment.

Style exaggeration. Similar to style removal, exaggerating all features in a shape can hardly lead to style exaggeration. The explicit knowledge that identifies style curves must be used here as well. Global approaches to feature exaggeration include frequency-domain amplification [VL08] and the application of a de-smoothing operator [SKB98]. As demonstrated in Figure 12, such approaches would not only exaggerate stylish features but also affect other parts of a shape. Having curve styles identified, the exaggeration can be confined to the style only. Our style exaggeration is implemented by enhancing the Laplacian coordinates [WCS*11] of each style curve V. Briefly, the new positions V' are found by solving the Poisson equations $LV' = wLV$, where w is a positive weight ($w > 1$ implies a style exaggeration and $w < 1$ corresponds to a depreciation).

Style blending. Using sets of shapes with many styles, new curve styles can be synthesized by style blending. Given two extracted style segments, we compute a blending between them by merging the Laplacian coordinates of the two style curves. To blend a source style curve C_s with a target style curve C_t, we first create a shared parameterization. We parameterize both by chord length and then re-sample C_s according the parameterization of C_t. Next, we blend C_s into C_t by calculating new positions V' for the vertices of C_t. The positions are found by solving a Poisson equations similar to the formulation used for style exaggeration:

$$L_t V' = w_s L_s V_s + w_t L_t V_t, \qquad (8)$$

where w_s and w_t are positive weights balancing the two styles, $w_s + w_t = 1$ and L_s and L_t are Laplacian matrices of C_s and C_t respectively (see Figure 13).

Style transfer. Transferring the style of shape O_s to shape O_t is more involved than blending style curves since these curves could be positioned in different places along the silhouette of the shapes. Hence, we first create a style-less base-shapes B_s and B_t by style removal. Next, we extract feature points from B_s and B_t using a similar method as in Section 4. The feature points partition both shapes to a set of curve segments which are delimited by a pair of adjacent feature points. Each such segment in B_s that occupied

Figure 12: From left to right in each triplet: the input shape, frequency-domain amplification result, and our style exaggeration result, respectively. Note how amplification creates a global change that may lead to distortions.

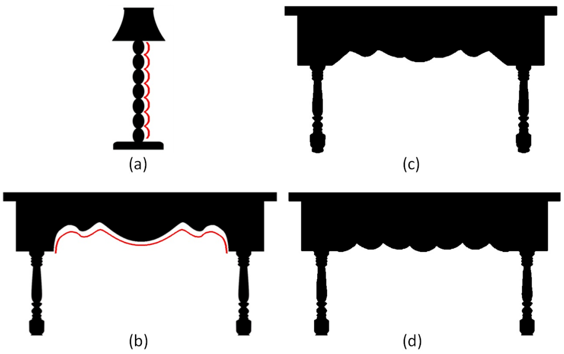


Figure 13: Blending a source style curve (a) into a target style curve (b). The style curves are marked by red. In (c) the blending weights are $w_s = w_t = 0.5$, and in (d) $w_s = 0.8$ $w_t = 0.2$.

an original segment with a style curve in O_s is defined as a *support region*. Next, we find the segment in B_t that best matches any support region in B_s, and copy its curve style. We use the dissimilarity measure defined in Equation (1). We also copy this style curve to any symmetric segments in B_t.

Style transfer leads to our key application based on style-content analysis: the synthesis of new shapes based on style transfer. We can synthesize missing shapes inside our style-content table by transferring the style of existing shapes in the table. Each missing entry, T_{ij}, is filled by transferring the style at row i to the shape in column j. Instead of using a single shape, we remove the style of all shapes in row i (defining the style), extract feature points and find candidate support regions. Using a style-less shape from column j (defining the content), we find the best matching support regions from all candidates and copy its style. Hence the style transfer is executed by blending the style curve of row i to the segment most similar to a support region in the target shape. For example, in Figure 14, 15, 16, the original shapes are shown in black and the new synthesized shapes created using style-transfer are shown in gold.

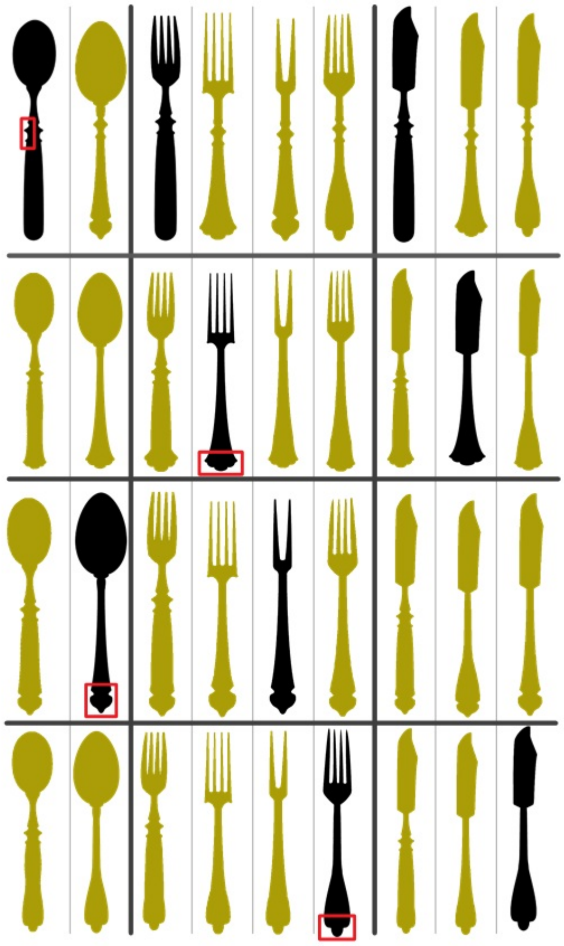


Figure 14: Synthesizing shapes using style transfer on a subset of cutlery: golden shapes are synthesized based on the black ones. Each row represents a style (marked by a red box), while contents are presented in hyper-columns, each specific shape occupies a sub-column.

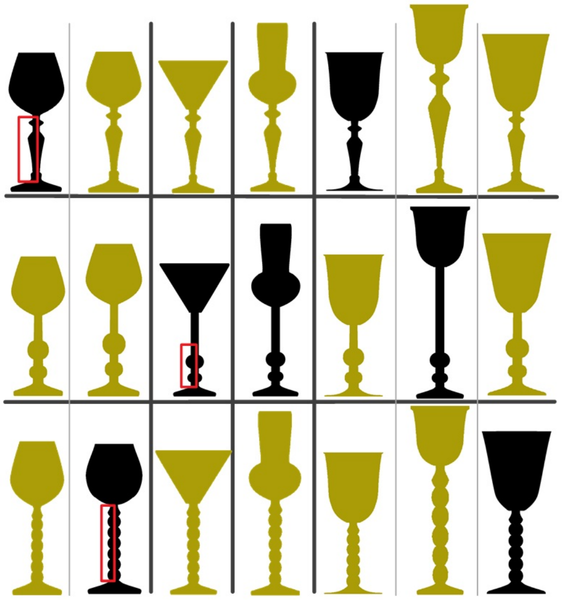


Figure 15: Synthesis by style transfer on a goblet set.

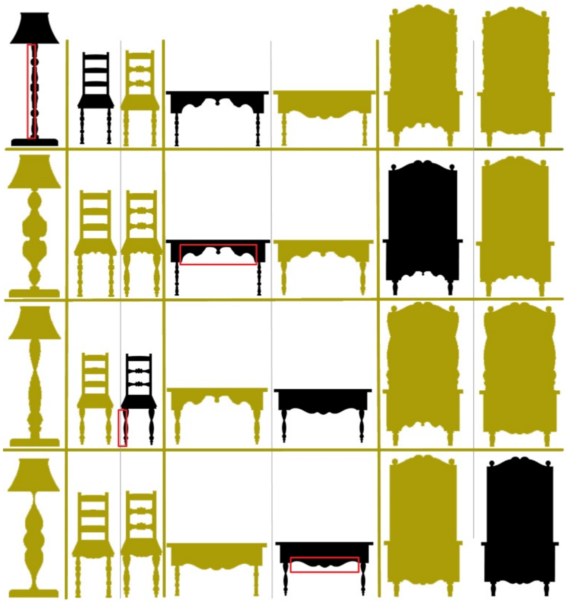


Figure 16: Synthesis by style transfer on a furniture set.

8. Discussion, limitation, and future work

The presented work is a preliminary attempt to answer the difficult and still open question of what makes a shape style and how to identify it. Our algorithm is unsupervised and lying at its core is a novel formulation for style-content separation in a set of shapes based on an analysis of feature-shape association. Results on several datasets demonstrate intuitive identification of local and decorative curve styles in a set of shapes. Several style-related applications are also developed to show the utility of curve style identification. Figure 17 offers a glimpse of the difficulty and perhaps also the beauty of the general problem of style extraction.

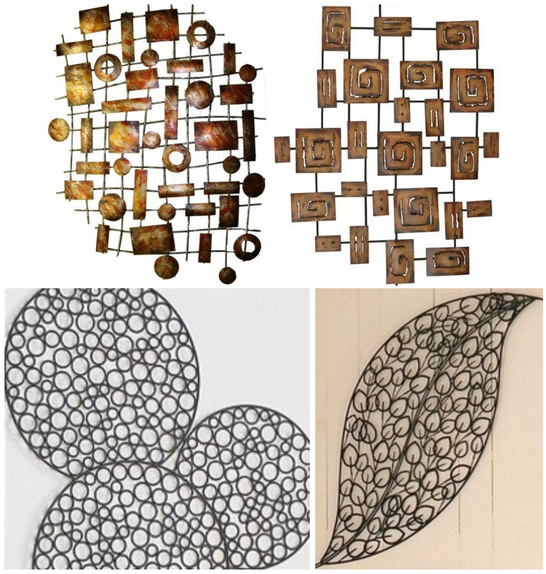


Figure 17: A set of metal wall art pieces which would make style analysis both interesting and challenging.

Generality. We believe the utility of the feature-shape association matrix and our formulation for style-content separation in a set is general. Substituting feature modes given by curve segments by other types of features allows other styles to be identified. However, any attempt to handle different forms of styles in one shot would face the difficult question of how to select the right features.

Styles aplenty. Styles can be about appearance or form. Our curve styles belong to the former while the apparent styles in Figure 17 are more about structure. The word style can mean so many different things and for some of them, it even seems difficult to articulate what the styles are, let alone finding proper mathematical formulations for them.

Limitations. We only identify curve styles along 2D silhouette profiles that are local and decorative in nature, i.e., we do not handle global appearance or structural styles, nor 3D shape styles. As an unsupervised set analysis, its success depends on the set given and fulfillment of the associated assumptions. Currently, the analysis algorithm relies on several tuned parameters. Although all the experimental results shown were obtained using the same parameter setting, providing some level of robustness indication, the best results for new inputs may require changed parameters. On a technical level, our current choices for the style-oriented curve similarity measure and clustering distances are both rather basic. Generally, the robustness of our analysis should still be subjected to more rigorous and larger-scale testing. In particular, automatic, perhaps data-driven, setting of the different threshold parameters is worth investigating. Finally, we still lack a principled evaluation for the analysis results; it would seem that to truly test whether the styles extracted appeal to our intuition, user evaluation is called for.

Future work. In addition to addressing issues related to implementation and evaluation, we also would like to pursue a few other problems. One of them is the identification of non-local decorative curve styles. The most interesting and challenging pursuit is the modeling and identification of styles in general. Exploring the use of a supervised approach may be interesting. One problem instance is to learn an unknown style from a set with the knowledge that the set indeed possesses some common styles. Moving from appearance-oriented styles to structural styles such as those revealed in Figure 17 is also interesting and challenging. Last but not the least, we would like to extend our style analysis to 3D shapes.

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